Spaces of σ-finite linear measure

I. Stasyuk, E.D. Tymchatyn*

September 21, 2012

Abstract

Spaces of finite n-dimensional Hausdorff measure are an important generalization of n-dimensional polyhedra. Continua of finite linear measure (also called continua of finite length) were first characterized by Eilenberg in 1938. It is well-known that the property of having finite linear measure is not preserved under finite unions of closed sets. Mauldin proved that if X is a compact metric space which is the union of finitely many closed sets each of which admits a σ-finite linear measure then X admits a σ-finite linear measure. We answer in the strongest possible way a 1989 question (private communication) of Mauldin. We prove that if a separable metric space is a countable union of closed subspaces each of which admits finite linear measure then it admits σ-finite linear measure. In particular, it can be embedded in the 1-dimensional Nöbeling space ν_1^3 so that the image has σ-finite linear measure with respect to the usual metric on ν_1^3.

1 Introduction

Eilenberg and Harrold [6] asked for a characterization of continua admitting finite n-dimensional Hausdorff measure. They obtained a number of characterizations of continua of finite linear measure. Most useful for us they proved that a space X admits a finite linear Hausdorff measure if and only if it is totally regular i.e. for each $x \in X$ and for each neighbourhood U of x there exist uncountably many nested neighbourhoods $\{U_\alpha\}$ of x with $U_\alpha \subset U$ such that $\mathrm{Bd}(U_\alpha) \cap \mathrm{Bd}(U_\beta) = \varnothing$ for $\alpha \neq \beta$ and with $\mathrm{Bd}(U_\alpha)$ finite. In particular, X is hereditarily locally connected, i.e., each connected subset of X is locally connected.

All spaces in this paper are separable and metric. We let $(\mathbb{R}^3, d)$ denote the Euclidean 3-space with its usual metric.

*The first named author was supported by Ontario MRI Postdoctoral fellowship at Nipissing University for advanced study and research in Mathematics. The second named author was supported in part by NSERC grant No. OGP 0005616.

2 Preliminaries

Definition 1. *Let (X, ρ) be a separable metric space and $\alpha \geq 0$. Then the α-dimensional Hausdorff measure H^α_ρ on X is defined by*

$$H^\alpha_\rho(A) = \sup_{\delta>0} \inf \left\{ \sum_{i=1}^{\infty} (\mathrm{diam}_\rho(U_i))^\alpha \mid A \subset \bigcup_{i=1}^{\infty} U_i \subseteq X,\ \mathrm{diam}_\rho(U_i) < \delta \text{ for every } i \in \mathbb{N} \right\}$$

for any $A \subset X$. We call H^1_ρ the linear Hausdorff measure on (X, ρ).

Definition 2. *The n-dimensional Nöbeling space ν^{2n+1}_n is the subspace of the Euclidean space $\mathbb{R}^{2n+1}$ which consists of all points with at most n rational coordinates.*

The space ν^{2n+1}_n is universal for separable metric spaces of dimension at most n.

Fremlin [7, Theorem 5H] proved that if Y is a space of finite linear measure then Y embeds in $(\mathbb{R}^3, d)$ so its image has finite linear measure with respect to the metric d. We shall need the following strengthening of Fremlin's result:

Theorem 1. *Let Y be a space which admits a metric ρ such that $H^1_\rho(Y) < \infty$. Then Y embeds in a continuum $X \subset \nu^3_1$ so $H^1_d(X) < \infty$.*

Proof. By [3], the space Y is totally regular. Let Z be the Freudenthal compactification of Y (see [8, page 109]). Then Z is a totally regular, metric compactum because finite separators of Y separate the distinct points of Z. The components of Z form a null-family of locally connected continua. By a standard argument one can adjoin to Z a countable null sequence of arcs to obtain a totally regular metric continuum X which contains Z. By [2, Theorem 3] the space X is the inverse limit of an inverse sequence (X_n, f^{n+1}_n) of finite connected graphs and monotone, surjective bonding maps so that each $f^{n+1}_n \colon X_{n+1} \to X_n$ has at most one non-degenerate fiber.

Represent the 1-dimensional Nöbeling space ν^3_1 as $\mathbb{R}^3 \setminus (\cup_{i=1}^{\infty} A_i)$ where each A_i is a straight line in $\mathbb{R}^3$ with each point of A_i having at least two rational coordinates. We equip ν^3_1 with the restriction of the usual metric d from $\mathbb{R}^3$.

We may assume that X and $\cup_{n=1}^{\infty} X_n$ are embedded in ν^3_1 so that X is also the limit of the sequence $\{X_n\}$ in the Hausdorff metric generated by d. Indeed, suppose that X_1 is embedded as a polygonal graph in ν^3_1. Let ε_1 be less than half the distance from the compact set X_1 to A_1. Assume that n is a positive integer and that $X_1, \ldots, X_n$ are embedded as polygonal graphs in ν^3_1 and $\varepsilon_1 > 2\varepsilon_2 > \cdots > 2^{n-1}\varepsilon_n$ are positive numbers such that for all $1 \leq i \leq n-1$,

1) $|H^1_d(X_{i+1}) - H^1_d(X_i)| < 2^{-i-1}$.

2) the Hausdorff distance from X_{i+1} to X_i is less than 2^{-i-2}.

3) the non-degenerate fiber of f^{i+1}_i has length less than 2^{-i-1}.

4) f^{i+1}_i is the identity off of a sufficiently small neighbourhood of the non-degenerate element of f^{i+1}_i.

5) the distance from X_i to A_j is greater than $2\varepsilon_j$ for $j \leq i \leq n$.

Let $\varepsilon_{n+1} > 0$ be smaller than $\frac{1}{2}\min\{\varepsilon_n, d(X_1 \cup \cdots \cup X_n, A_1 \cup \cdots \cup A_{n+1})\}$. We may take X_{n+1} to be a polygonal graph in ν_1^3 so that conditions 1)-5) are satisfied for $1 \leq i \leq n$. It follows that the sequence $\{X_n\}$ converges to X in the Hausdorff metric. By 5), $X \subset \nu_1^3$ and $H_d^1(X) = \lim_{i\to\infty} H_d^1(X_i) < \infty$ by 1) and 4). □

It follows trivially from Theorem 1 that every space of finite length has a compactification of finite length in (ν_1^3, d).

Definition 3. *A closed subset A of a complete metric space Y is called a Z-set if for each open cover $\mathcal{U}$ of Y there is a function $f\colon Y \to Y \setminus A$ which is $\mathcal{U}$-close to* Id_Y *i.e. for every $y \in Y$ there is $U \in \mathcal{U}$ with $y, f(y) \in U$. If the map f can be chosen in such a way that $\overline{f(Y)} \cap A = \varnothing$ then A is called a strong Z-set.*

Definition 4. *For a space A and a complete metric space Y an embedding $g\colon A \to Y$ is called a Z-embedding if its image is a Z-set in Y.*

Definition 5. *Let Y and Z be topological spaces and let $C(Y, Z)$ denote the set of all continuous functions from Y to Z. For each map $f: Y \to Z$ and for each open cover $\mathcal{S}$ of Z we let $B(f, \mathcal{S})$ denote the set of all maps in $C(Y, Z)$ that are $\mathcal{S}$-close to f. Define a collection $\mathcal{T}$ of subsets of $C(Y, Z)$ by the rule: a subset $U \subset C(Y, Z)$ is an element of $\mathcal{T}$ if for every $f \in U$, there exists an open cover $\mathcal{U}$ of Z such that $B(f, \mathcal{U}) \subset U$. If U and V are elements of $\mathcal{T}$ such that $B(f, \mathcal{U}) \subset U$ and $B(f, \mathcal{V}) \subset V$ for open covers $\mathcal{U}$ and $\mathcal{V}$ of Z, then $B(f, \mathcal{W}) \subset U \cap V$ for any open cover $\mathcal{W}$ which refines both $\mathcal{U}$ and $\mathcal{V}$. The collection $\mathcal{T}$ is called the limitation topology on $C(Y, Z)$.*

It is known that the limitation topology coincides with the topology of uniform convergence with respect to all compatible metrics on Y and Z (see [4, Lemma 2.1.4]).

Definition 6. *Let n be a positive integer. A Polish space Y is called an absolute (neighbourhood) extensor in dimension n, or shortly, an $A(N)E(n)$-space, if any map $f\colon A \to Y$, defined on a closed subspace A of a Polish space B with $\dim B \leq n$, can be extended to a map of the space B (respectively, of a neighbourhood of A in B) into Y.*

Definition 7. *A Polish space Y is called strongly $\mathcal{A}_{\omega,n}$-universal if any map of any at most n-dimensional Polish space into Y can be arbitrarily closely approximated by closed embeddings.*

We will need the following result (see [4, Proposition 5.1.7]).

Proposition 1. *Let Y be an at most n-dimensional strongly $\mathcal{A}_{\omega,n}$-universal Polish $ANE(n)$-space, and A a closed subspace of an at most n-dimensional Polish space B. Then each map $f\colon B \to Y$, such that the restriction $f|_A$ is a Z-embedding, can be arbitrarily closely approximated by Z-embeddings coinciding with f on A. In particular, the set of all Z-embeddings of B into Y is a dense G_δ subset of $C(B, Y)$.*

It is known that the n-dimensional Nöbeling space ν_n^{2n+1} is a strongly $\mathcal{A}_{\omega,n}$-universal, $ANE(n)$-space. The following three statements are proved in [5] as Proposition 3.6, Lemma 3.2 and Proposition 3.8, respectively.

Proposition 2. *Let P be an at most n-dimensional Polish space and let $C(P, \nu_n^{2n+1})$ denote the set of all continuous functions from P into ν_n^{2n+1} with the limitation topology. Then the set of all Z-embeddings of P into ν_n^{2n+1} is a dense G_δ subset of $C(P, \nu_n^{2n+1})$.*

Proposition 3. *Each compact subset of ν_n^{2n+1} is a strong Z-set.*

Proposition 4. *Each homeomorphism between compact subsets of ν_n^{2n+1} can be extended to an autohomeomorphism of ν_n^{2n+1}.*

Definition 8. *A point x of a connected space X is a local cut point of X if it disconnects some connected neighbourhood of x. The local cut point x is said to be of order 2 in X if it has a basis of neighbourhoods with two point boundaries.*

Theorem 2. *If X is a connected and totally regular space then X has at each point an uncountable local basis of open sets $\{U_\alpha\}$ with finite boundaries and such that each boundary point of U_α is a point of order 2 in X.*

Proof. Let Y be a totally regular continuum containing X and constructed as in the proof of Theorem 1. Each local cut point of Y is a local cut point of X. By [10, III, 9.2] all but at most countably many local cut points of Y are of order 2 in Y. □

Definition 9. *We say that a space Y admits σ-finite linear measure if there is a metric ρ on Y and a family $\{A_i\}_{i=1}^\infty$ of closed subsets of Y so $Y = \cup_{i=1}^\infty A_i$ and $H_\rho^1(A_i) < \infty$ for each i.*

Theorem 3 ([9])**.** *Let Y be a compact metric space. If Y may be expressed as $\cup_{i=1}^n M_i$ where each M_i is closed and admits σ-finite linear measure, then Y admits σ-finite linear measure.*

3 Main result

Theorem 4. *Let $X = \bigcup_{i=1}^\infty X_i$ where each X_i is totally regular and closed in X. Then the space X can be embedded in ν_1^3 so that the image of X has σ-finite linear measure with respect to the usual metric d on ν_1^3.*

Proof. Let $h_1'\colon X_1 \to \tilde{X}_1 \subset \nu_1^3$ be a compactification of X_1 where $\tilde{X}_1$ has finite length with respect to the metric d by Theorem 1. By Proposition 1, the map h_1' extends to an embedding $h_1\colon X \to \nu_1^3$ of X since ν_1^3 is an $\mathcal{A}_{\omega,1}$ space.

Let $\mathcal{U}_1'$ be a locally finite cover of $\mathbb{R}^3 \setminus \tilde{X}_1$ by open topological 3-balls such that $\operatorname{diam}_d(U') < \min\{1/4, d(\tilde{X}_1, U')/4\}$ for each $U' \in \mathcal{U}_1'$. We denote by $\mathcal{U}_1$ the cover of $\nu_1^3 \setminus \tilde{X}_1$ which is induced by $\mathcal{U}_1'$, i.e. $\mathcal{U}_1 = \{U' \cap \nu_1^3 \mid U' \in \mathcal{U}_1'\}$. Let $\mathcal{V}_2 = \{X_{2,1}, X_{2,2}, \dots\}$ be a locally finite in $\mathbb{R}^3 \setminus \tilde{X}_1$ closed cover of $h_1(X_2) \setminus \tilde{X}_1$ and let $\{I_{2,1}, I_{2,2}, \dots\}$ be finite sets of local cutpoints of order 2 in $h_1(X_2)$ such that

$$I_{2,i} \subset X_{2,i},\ X_{2,i} \cap X_{2,j} \subset I_{2,i} \cap I_{2,j} \text{ for } i \neq j,\ \bigcup_{i=1}^\infty I_{2,i} \text{ is discrete in } \mathbb{R}^3 \setminus \tilde{X}_1$$

and such that $\mathcal{V}_2$ refines $\mathcal{U}_1$. For each i let $U_{2,i} \in \mathcal{U}_1$ so $X_{2,i} \subset U_{k,i}$. For each i let $T_{2,i} \subset U_{2,i}$ be a polygonal tree in ν_1^3 with set of endpoints $I_{2,i}$ such that $T_{2,i} \cap T_{2,j} = I_{2,i} \cap I_{2,j}$. For each i let $W_{2,i} = W_{2,i}' \cap \nu_1^3$ where $W_{2,i}'$ is a closed polyhedral 3-ball in $\mathbb{R}^3$ such that

$$T_{2,i} \subset W_{2,i} \subset U_{2,i},\ T_{2,i} \cap \operatorname{Bd}(W_{2,i}) = I_{2,i} \text{ and } W_{2,i} \cap W_{2,j} = I_{2,i} \cap I_{2,j} \text{ for } i \neq j.$$

For each i let $h_{2,i}\colon X_{2,i} \to \tilde{X}_{2,i} \subset \operatorname{Int}_{\nu_1^3}(W_{2,i}) \cup I_{2,i}$ be a compactfication where each $\tilde{X}_{2,i}$ has finite length with respect to the metric d and $h_{2,i}|_{I_{2,i}} = \operatorname{Id}_{I_{2,i}}$. Let $\tilde{X}_2 = \bigcup_{i=1}^\infty \tilde{X}_{2,i}$. Note that $\tilde{X}_1 \cup \tilde{X}_2$ is compact. Let $h'_2 \colon \tilde{X}_1 \cup h_1(X_2) \to \tilde{X}_1 \cup \tilde{X}_2$ be the embedding such that $h'_2|_{\tilde{X}_1} = \mathrm{Id}_{\tilde{X}_1}$ and $h'_2|_{X_{2,i}} = h_{2,i}$ for all i. Note that $h'_2|_{h_1(X_2 \setminus X_1)}$ is $\mathcal{U}_1$-close to $h_1|_{X_2 \setminus X_1}$.

Since $\tilde{X}_1 \cup \tilde{X}_2$ is compact in ν^3_1, it is a strong Z-set, and so h'_2 can be extended to an embedding h_2 of $\tilde{X}_1 \cup h_1(X)$ such that $h_2|_{h_1(X) \setminus \tilde{X}_1}$ is $\mathcal{U}_1$-close to $h_1|_{X \setminus X_1}$.

Let $\mathcal{U}'_2$ be a locally finite cover of $\mathbb{R}^3 \setminus (\tilde{X}_1 \cup \tilde{X}_2)$ by open topological 3-balls such that $\mathrm{diam}_d(U') < \min\{1/8, d(\tilde{X}_1 \cup \tilde{X}_2, U')/8\}$ for $U' \in \mathcal{U}'_2$. We denote by $\mathcal{U}_2$ the cover of $\nu^3_1 \setminus (\tilde{X}_1 \cup \tilde{X}_2)$ which is induced by $\mathcal{U}'_2$, i.e. $\mathcal{U}_2 = \{U' \cap \nu^3_1 \mid U' \in \mathcal{U}'_2\}$.

Suppose now that for $1 \le n \le k-1$ the covers $\mathcal{U}_n$, the spaces $\tilde{X}_n$ and the embeddings h_n are defined so that the following conditions are satisfied:

1) $\tilde{X}_1 \cup \cdots \cup \tilde{X}_n$ is compact and of σ-finite linear measure in (ν^3_1, d),

2) $\mathcal{U}_n$ is a cover of $\nu^3_1 \setminus (\tilde{X}_1 \cup \cdots \cup \tilde{X}_n)$ induced by a locally finite cover of $\mathbb{R}^3 \setminus (\tilde{X}_1 \cup \cdots \cup \tilde{X}_n)$ by open topological 3-balls such that

$$\mathrm{diam}_d(U) < \min\{2^{-n-1}, 2^{-n-1} d(\tilde{X}_1 \cup \cdots \cup \tilde{X}_n, U)\}$$

for each $U \in \mathcal{U}_n$,

3)

$$h_n \colon h_{n-1} \circ \cdots \circ h_1(X) \cup \tilde{X}_1 \cup \cdots \cup \tilde{X}_{n-1} \to \nu^3_1$$

is an embedding such that

$$h_n|_{\tilde{X}_1 \cup \cdots \cup \tilde{X}_{n-1}} = \mathrm{Id}_{\tilde{X}_1 \cup \cdots \cup \tilde{X}_{n-1}} \quad \text{and}$$

$$h_n|_{h_{n-1} \circ \cdots \circ h_1(X) \setminus (\tilde{X}_1 \cup \cdots \cup \tilde{X}_{n-1})} \text{ is } \mathcal{U}_{n-1}\text{-close to } h_{n-1}|_{h_{n-2} \circ \cdots \circ h_1(X) \setminus (\tilde{X}_1 \cup \cdots \cup \tilde{X}_{n-1})}.$$

Let $\mathcal{V}_k = \{X_{k,1}, X_{k,2}, \dots\}$ be a locally finite in $\mathbb{R}^3 \setminus (\tilde{X}_1 \cup \cdots \cup \tilde{X}_{k-1})$ closed cover of $h_{k-1} \circ \cdots \circ h_1(X_k) \setminus (\tilde{X}_1 \cup \cdots \cup \tilde{X}_{k-1})$ and let $\{I_{k,1}, I_{k,2}, \dots\}$ be finite sets of local cutpoints of order 2 in $h_{k-1} \circ \cdots \circ h_1(X_k)$ such that

$$I_{k,i} \subset X_{k,i}, \ X_{k,i} \cap X_{k,j} = I_{k,i} \cap I_{k,j} \text{ for } i \ne j,$$

$$\bigcup_{i=1}^{\infty} I_{k,i} \text{ is discrete in } \mathbb{R}^3 \setminus (\tilde{X}_1 \cup \cdots \cup \tilde{X}_{k-1})$$

and such that $\mathcal{V}_k$ refines $\mathcal{U}_{k-1}$. For each i let $U_{k,i} \in \mathcal{U}_{k-1}$ such that $X_{k,i} \subset U_{k,i}$ and let $T_{k,i}$ be a polygonal tree in $U_{k,i}$ with set of endpoints $I_{k,i}$ with $T_{k,i} \cap T_{k,j} = I_{k,i} \cap I_{k,j}$. For each i let $W_{k,i} = W'_{k,i} \cap \nu^3_1$ where $W'_{k,i}$ is a closed polyhedral 3-ball in $\mathbb{R}^3$ such that

$$T_{k,i} \subset W_{k,i} \subset U_{k,i}, \ T_{k,i} \cap \mathrm{Bd}(W_{k,i}) = I_{k,i}, \text{ and } W_{k,i} \cap W_{k,j} = I_{k,i} \cap I_{k,j} \text{ for } i \ne j.$$

For each i let $h_{k,i} \colon X_{k,i} \to \tilde{X}_{k,i} \subset \mathrm{Int}_{\nu^3_1}(W_{k,i}) \cup I_{k,i}$ be a compactification where $X_{k,i}$ has finite length with respect to d be such that $h_{k,i}|_{I_{k,i}} = \mathrm{Id}_{I_{k,i}}$. Let $\tilde{X}_k = \bigcup_{i=1}^{\infty} \tilde{X}_{k,i}$ and let

$$h'_k \colon \tilde{X}_1 \cup \cdots \cup \tilde{X}_{k-1} \cup h_{k-1} \circ \cdots \circ h_1(X_k) \to \tilde{X}_1 \cup \cdots \cup \tilde{X}_k$$

be the compactification such that

$$h'_k|_{\tilde{X}_1 \cup \cdots \cup \tilde{X}_{k-1}} = \mathrm{Id}_{\tilde{X}_1 \cup \cdots \cup \tilde{X}_{k-1}} \text{and } h'_k|_{X_{k,i}} = h_{k,i} \ \text{ for each } i.$$

Note that

$$h'_k|_{h_{k-1}\circ\cdots\circ h_1(X_k\setminus(X_1\cup\cdots\cup X_{k-1}))}$$

is $\mathcal{U}_{k-1}$-close to $h_{k-1}\circ\cdots\circ h_1|_{(X_k\setminus(X_1\cup\cdots\cup X_{k-1}))}$.

Since $\tilde{X}_1\cup\cdots\cup\tilde{X}_k$ is compact in ν_1^3, h'_k can be extended to an embedding h_k of $\tilde{X}_1\cup\cdots\cup\tilde{X}_{k-1}\cup h_{k-1}\circ\cdots\circ h_1(X)$ such that

$$h_k|_{h_{k-1}\circ\cdots\circ h_1(X)\setminus(\tilde{X}_1\cup\cdots\cup\tilde{X}_{k-1})} \text{ is } \mathcal{U}_{k-1}\text{-close to } h_{k-1}|_{h_{k-2}\circ\cdots\circ h_1(X)\setminus(\tilde{X}_1\cup\cdots\cup\tilde{X}_{k-1})}.$$

Let $\mathcal{U}'_k$ be a locally finite cover of $\mathbb{R}^3\setminus(\tilde{X}_1\cup\cdots\tilde{X}_k)$ by open polyhedral 3-balls so $\mathrm{diam}_d(U)<\min\{2^{-k-1},2^{-k-1}d(\tilde{X}_1\cup\cdots\cup\tilde{X}_k,U)\}$ for each $U\in\mathcal{U}'_k$ and let $\mathcal{U}_k$ be the corresponding induced cover of $\nu_1^3\setminus(\tilde{X}_1\cup\cdots\cup\tilde{X}_k)$.

Then by induction h_k is defined for each positive integer k. Let $h=\lim_{k\to\infty}h_k\circ\cdots\circ h_1$. Since the sequence $\{h_k\circ\cdots\circ h_1\}_{k=1}^{\infty}$ is uniformly convergent, h is a continuous function. Since every function $h_k\circ\cdots\circ h_1$ is one-to-one, for each $x\in X$ there exists a positive integer n such that $x\in X_n$ so $h_k\circ\cdots\circ h_n\circ\cdots\circ h_1(x)=h_n\circ\cdots\circ h_1(x)$ for $k\geq n$. It follows that h is one-to-one. If $x\in X\setminus(X_1\cup\cdots\cup X_k)$ and $h_k\circ\cdots\circ h_1(x)\in U\in\mathcal{U}_{k-1}$ then

$$h(x)\in \mathrm{St}^2(U,\mathcal{U}_{k-1})\subset\overline{\mathrm{St}^2(U,\mathcal{U}_{k-1})}\subset h(X)\setminus(\tilde{X}_1\cup\cdots\cup\tilde{X}_k)$$

as in [1, Theorem 4.2]. Hence, h is open. Thus, h is an embedding of X into $\cup_{i=1}^{\infty}\tilde{X}_i$. The space $\cup_{i=1}^{\infty}\tilde{X}_i$ is σ-compact and of σ-finite linear measure. □

Note. Theorem 4 is sharp in the following sense. It is not true that a space of σ-finite linear measure embeds in a compact space of σ-finite linear measure. For if $X=\mathbb{Q}\times[0,1]$ where $\mathbb{Q}$ is the space of rational numbers then X has σ-finite linear measure. It is easy to see that if $\tilde{X}$ is a metric compactification of X then each separation of $\tilde{X}$ between $(0,0)$ and $(0,1)$ contains a perfect set. However, Mauldin has shown that a space with σ-finite linear measure has a basis of open sets with countable boundaries.

References

[1] R. D. Anderson and R. H. Bing, *A complete elementary proof that Hilbert space is homeomorphic to the countable infinite product of lines*, Bull. Amer. Math. Soc. **74** (1968), 771–792.

[2] R. Buskirk, J. Nikiel, E.D. Tymchatyn, *Totally regular curves as inverse limits*, Houston J. Math. **18** (1992), no. 3, 319–327.

[3] S. Eilenberg, *On continua of finite length*, Ann. Soc. Polon. Math. **17** (1938), 253–254.

[4] A. Chigogidze, Inverse Spectra, North-Holland, Amsterdam (1996).

[5] A. Chigogidze, K. Kawamura and E. D. Tymchatyn, *Nöbeling spaces and the pseudo-interiors of Menger compacta*, Topology Appl. **68** (1996), 33-65.

[6] S. Eilenberg and O. G. Harrold Jr., *Continua of finite linear measure I*, Amer. J. Math. **65** (1943), 137–146.

[7] D.H. Fremlin, *Spaces of finite length*, Proc. London Math. Soc. **64** (1992), 449–486.

[8] J. R. Isbell, Uniform spaces, Amer. Math. Soc., Providence, 1964.

[9] R. D. Mauldin, Continua with σ-finite linear measure, in Measure Theory, Oberwolfach 1990, Proceedings of the Conference held at Oberwolfach, March 18-24, 1990 Supplemento di Rendiconti del Circolo Matematico di Palermo, serie II, no. 38, 1992..

[10] G.T. Whyburn, Analytic Topology, Amer. Math. Soc. Colloq. Publ 28, Providence, 1942.

I. Stasyuk
Department of Computer Science and Mathematics, Nipissing University,
100 College Drive, Box 5002, North Bay, ON, 51B 8L7, Canada
e-mail address ihors@nipissingu.ca, i_stasyuk@yahoo.com

E.D. Tymchatyn
Department of Mathematics and Statistics, McLean Hall, University of Saskatchewan,
106 Wiggins Road, Saskatoon, SK, S7N 5E6, Canada
e-mail address tymchat@math.usask.ca